The Sound Intensity probe

The intensity-measurement technique is a powerful tool for locating sound sources, order rank them and determine the emitted sound power. The traditional method is based on the simultaneous determination of sound pressure and particle velocity by two closely spaced microphones. The sound intensity probe must ensure a well-defined acoustic spacing between the microphones as well as minimise disturbances to the sound field.

Figure 1: A p-p Sound Intensity probe (Courtesy Bruel & Kjaer).

To ensure maximum measurement accuracy, the spacing between the microphones must be optimised for the given measurement conditions. At low frequencies, in highly reverberant conditions, this spacing must be large, whereas for measurements at high frequencies, it must be small.

The dominating method of measuring sound intensity in air is based on the combination of two pressure microphones, as shown above. However, a sound intensity probe that combines an acoustic particle velocity transducer with a pressure microphone has recently become available.

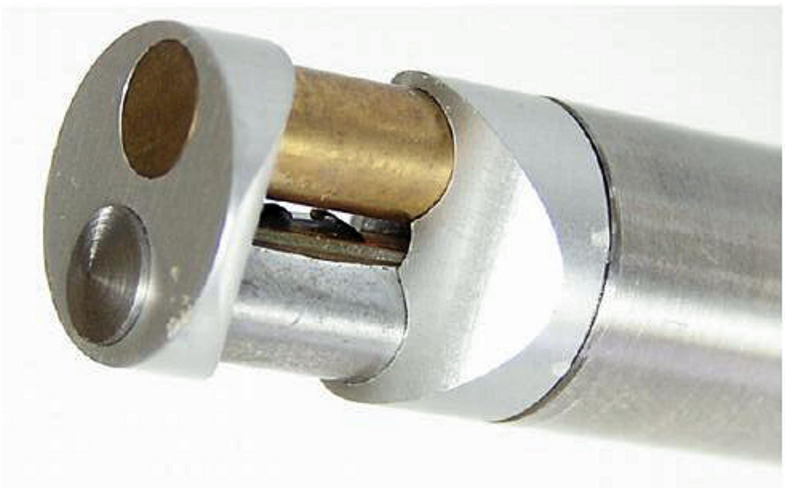


Figure 2: A p-v Sound Intensity probe (Courtesy Microflown).

When a p-p probe is employed, the signals coming from the two pressure microphones require some processing, for retrieving the p-v signals (sound pressure and particle velocity).

The basis of the method is to use the 1st Newton's law for relative the spatial pressure gradient to the time derivative of particle velocity

1st Newton's law states that:

$$F = m \cdot a = m \cdot \frac{dv}{d\tau}$$

If we apply this equation to a small volume of air, in the shape of a small parallelepiped dx·dy·dz, and we only consider the x direction, we see that the force F applied is given by the pressure difference on the opposite faces, multiplied by the area of these faces.

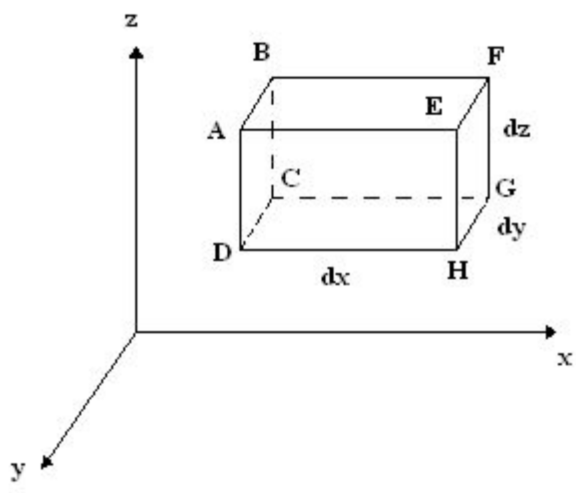


Figure 3: the infinitesimal volume dV

We can call dF the infinitesimal force applied to the small volume dV along the x axis:

$$dF = dp \cdot dy \cdot dz = \frac{dp}{dx} \cdot dx \cdot dy \cdot dz = \frac{dp}{dx} \cdot dV$$

On the other hand, also the mass of this very small volume is an infinitesimal mass dM:

$$dM = \rho \cdot dV$$

Hence the 1st Newton's equation becomes what's known as the Euler's equation:

$$dF = dM \cdot \frac{dv}{d\tau}$$

$$\frac{dp}{dx} \cdot dV = \rho \cdot dV \cdot \frac{dv}{d\tau}$$

$$\frac{dp}{dx} = \rho \cdot \frac{dv}{d\tau}$$

Hence, we can compute the component of the particle velocity along the x axis from the time-integral of the pressure gradient:

$$v_x = \frac{1}{\rho} \cdot \int \frac{dp}{dx} \cdot d\tau$$

We can approximate the pressure gradient with a finite-difference approximation, as we know the sound pressure by means of two microphones (M_1 and M_2) which are at a distance d, and which capture the signals p_1 and p_2:

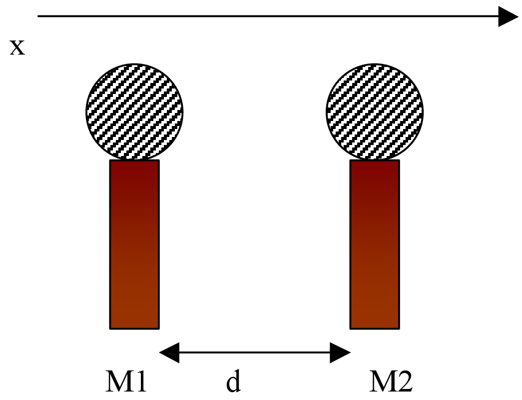


Figure 4: scheme of a p-p Sound Intensity probe

With the finite difference approximation, we can estimate the particle velocity v and the sound pressure p at the center of the probe as:

$$v = \frac{1}{\rho} \cdot \int \frac{p_1 - p_2}{d} \cdot d\tau$$

$$p = \frac{p_1 + p_2}{2}$$

And finally we can compute the Sound Intensity I and the Sound Energy Density D following their definitions (these also holds for a p-v probe, which already outputs electrical signals proportional to p and v):

$$I = \overline{p.v}$$

$$D = \frac{1}{2} \cdot \left[\rho \cdot v^2 + \frac{p^2}{\rho \cdot c_0{}^2} \right]$$

The following figures show how the finite difference approximation causes errors at low and high frequency: at low frequency the error is due to the phase mismatch between the two microphones, at high frequency the error is caused by the fact that the microphone spacing d becomes comparable with the wavelength λ.

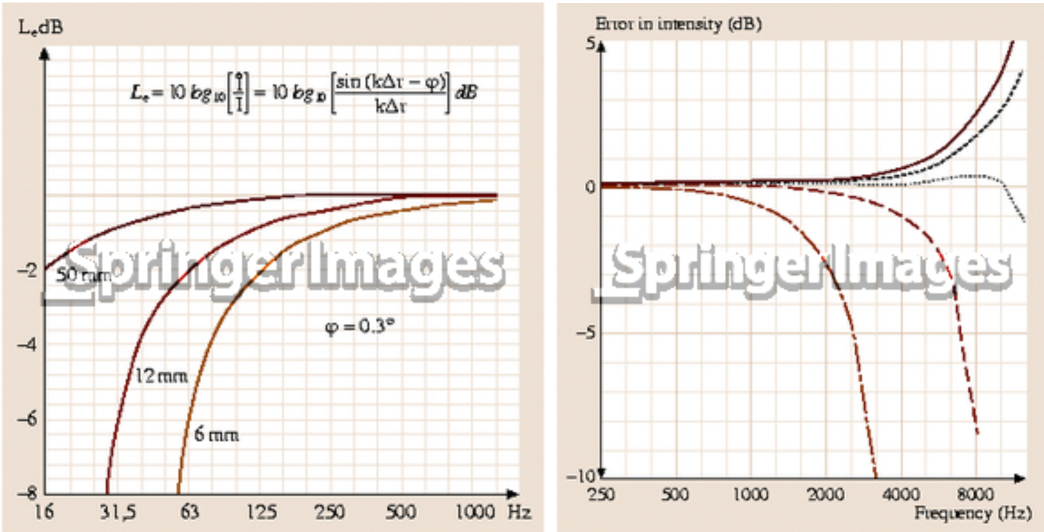


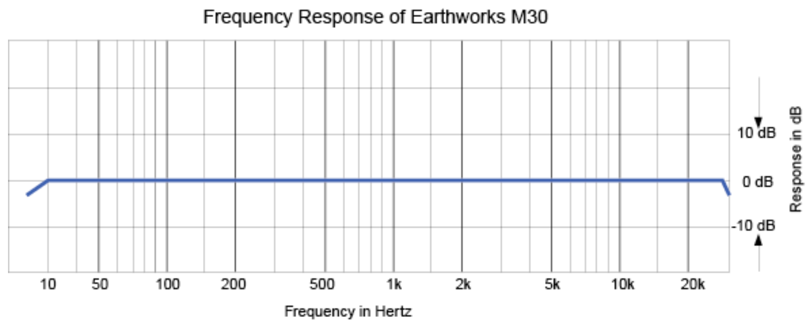
Figure 5: errors of a p-p Sound Intensity probe
for different microphone spacings

MICROPHONES

We analyze 3 different types of microphones.

Omnidirectional microphones

ISO3382 recommends the usage of omni mikes of not more than 13 mm diameter (small microphones). The frequency response of these microphones is absolutely flat: this is an ideal form.



Stereo microphones

For measuring "spatial acoustical parameters" such as LF or IACC, the usage of stereo (32-channels) microphone systems is quite common.

The initial approach was to use directive microphones for gathering some information about the spatial properties of the sound field "as perceived by the listener". Two different approaches emerged: binaural dummy heads and pressure-velocity microphones.

Binaural microphones that simulates the human ear, and in some case not only the head, but also the torso.

It is also possible to wear small in-ear microphones, so that a true human body is employed, instead of a “dummy” one:

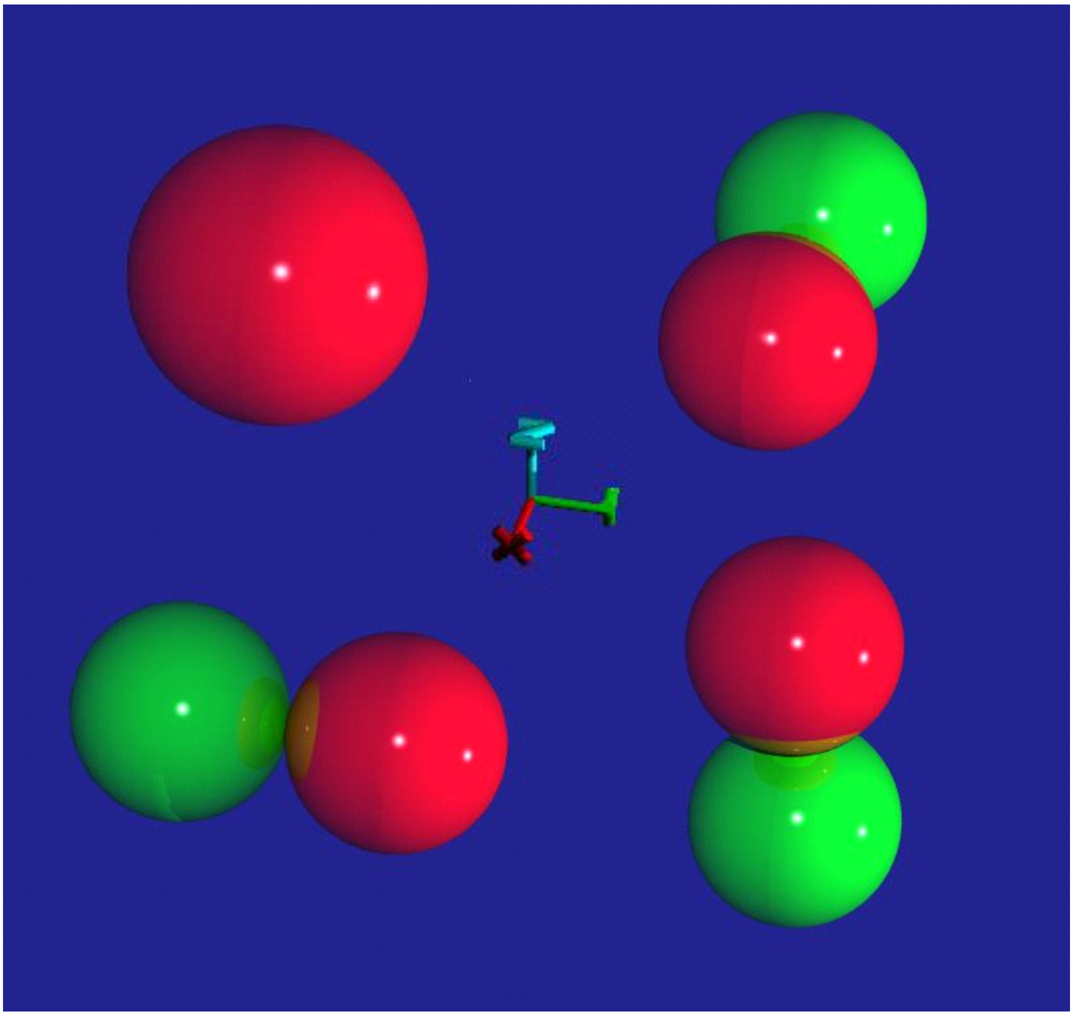
Binaural microphones are required for measuring the spatial parameter IACC
Another type of stereo microphones employed is a pressure-velocity microphone, which provides a directivity pattern which can be either omnidirectional or figure-of-eight:

Only a few of these microphones can simultaneously output the pressure (omnidirection) and particle velocity (figure-of—eight) signals: most of them, as the one shown above, only output one of the two polar patterns at once. So the measurement has to be repeated twice, after switching the directivity by rotating the ring of the microphone.

B-format (4-channels) microphones

Gerzon and Craven created and developed the Soundfield microphone, a tetrahedral microphone. This is its modern version:

This type of microphone allows to make simultaneous measurements of the omnidirectional pressure and of the 3 cartesian components of particle velocity.



Polar pattern of the 4 channels (W, X, Y, Z) of a Soundfield microphone, constituting the so-called B-format signal

The original Soundfield microphone did emply a small “black box”, containing complex analog electronic circuitry, for performing the required filtering. This converts the raw signals coming from the 4 capsules (“A-format”) in the proper B-format signals.

Outdoors propagation of spherical waves in free field

The D' Alembert equation

Sound is produced by a source and it's transmitted as propagation to a receiver. Propagation is a phenomenon which is just wave-like and that it could occur both outdoors and indoors.

In order to study the case of outdoors propagation in free field it has been introduced the so-called D' Alembert equation, which takes its name from the physicist, mathematician and French philosopher who lived between the first and the second half of eighteenth century and who computed the equation.
In particular it is a combination of the continuity equation for fluid motion and of the first Newton equation.
First, in order to write it, let's consider the Euler's equation:

$$grad(p) = \rho \cdot \frac{\partial v}{\partial \tau} \qquad (1)$$

Next we can introduce a scalar variable, that is called potential ***Φ*** of the acoustic field, which represents a sort of "common base" of sound pressure ***p*** and particle velocity ***v***. Indeed we can note that the spatial and temporal gradient of the potential gives exactly the sound pressure and the particle velocity, as shown here:

$$p = -\rho_0 \cdot \frac{\partial \phi}{\partial \tau} \qquad \vec{v} = grad(\phi) \qquad (2)$$

Finally it's enough substituting these two identities in Euler's equation to get the following formula:

D 'Alembert equation

$$\frac{\partial^2 \phi}{\partial \tau^2} = c^2 \cdot \nabla^2 \phi \qquad (3)$$

The D' Alembert equation permits us to study sound wavefronts propagation in every point of a free field.

Integrating this formula, in fact, we can get the potential ***Φ*** and of consequence the pressure and velocity field.
Unfortunately this solving method results so much mathematically hard so we can get solutions only in a few cases. This is possible for progressive plane waves, standing plane waves and spherical waves radiated by a point source.

Solutions of D' Alembert equation for spherical waves

Let's consider a sound point source that produces spherical waves.
These waves are generated by a pulsating sphere of radius **R**, even called "monopole" source, which is represented in *Figure 1*.

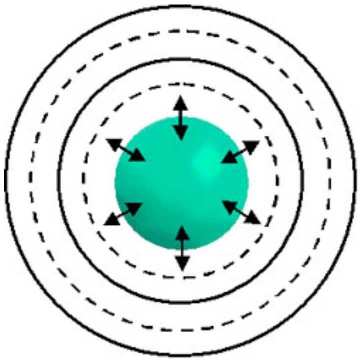


Fig.1 –Schematic representation of a pulsating sphere

This one is defined "pulsating" due to its continuous and periodical expansions and compressions.
Furthermore it's possible supposing to know and to define two quantities related to this sphere:

- **Volume velocity** or **Source strength** (where **S** is the area of the spherical surface):

$$Q = v_{max} \cdot S \qquad (4)$$

- **Radial velocity of the sphere's surface** (where $\omega = 2\pi f$ is the angular velocity):

$$v(R,\tau) = v_{max} \cdot \cos(\omega\tau) \qquad (5)$$

This formula could be written as in the following expression::

$$v(R,\tau) = v_{max} \cdot e^{j\omega\tau} \qquad (6)$$

where $e^{j\omega\tau} = \cos(\omega\tau) + j\sin(\omega\tau)$
$\cos(\omega\tau)$: *real part*
$j\sin(\omega\tau)$: *imaginary part*

In the previous equation we can note an imaginary exponential term, which represents the periodicity of the surface's velocity. This representation is acceptable because, as we can see above, it could be written as a sum of periodic functions. In particular the imaginary unit is a mathematic artifice which doesn't exist in the real world, where waves occur. Because of this it will be possible to neglect the fictitious imaginary part from the expression of the exponential term, obtaining back equation (5).

Finally we deduce that both (5) and (6) are the same in the real world, where imaginary numbers do not exist.

Now, known these quantities, we consider the pulsating sphere of radius R and in particular the outgoing waves, which means that $r \geq R$.

In this condition it's quite easy to solve the D' Alembert equation of the previous paragraph for spherical waves, extracting the value of particle velocity at any radius r:

Particle velocity

$$v(r,\tau) = v_{max} \cdot \frac{R^2}{r^2} \cdot \frac{1 + j \cdot k \cdot r}{1 + j \cdot k \cdot R} \cdot e^{j[\omega\tau - k(r-R)]} \qquad (7)$$

Then, thanks to Euler's equation, it's possible to compute the value of sound pressure:

Sound pressure

$$p(r,\tau) = \rho_0 \cdot c \cdot v_{max} \cdot \frac{j \cdot k \cdot R^2}{r \cdot (1 + j \cdot k \cdot R)} \cdot e^{j[\omega\tau - k(r-R)]} \qquad (8)$$

where in both cases $k = \frac{\omega}{c} = \frac{2\pi f}{c} = \frac{2\pi}{\lambda}$

In particular **k** is called the wave number.

Let's take now the particle velocity (7), and let's evaluate the modulus of the oscillating velocity:

$$|v(r,\tau)| = v_{max} \cdot \frac{R^2}{r^2} \cdot \sqrt{\frac{1 + (k \cdot r)^2}{1 + (k \cdot R)^2}} \qquad (9)$$

Looking at this formula it appears that the dependence of the modulus of particle velocity *v* from radius *r* follows a not-linear law, which can be approximated by a proportionality to *1/r*^{2} for small values of *kr*, and by a proportionality to *1/r* for large values of *kr*.

Effects and proprieties of spherical waves in free field

Proximity effect

The analysis of the solutions (8) and (9) of the D' Alembert equation can be studied in two different kinds of sound field: the far field and the near field. These ones are not absolute definitions as meaning that, in any case, the real distance between source and receiver couldn't be the only value to be considered, but it must be compared to the wavelength.

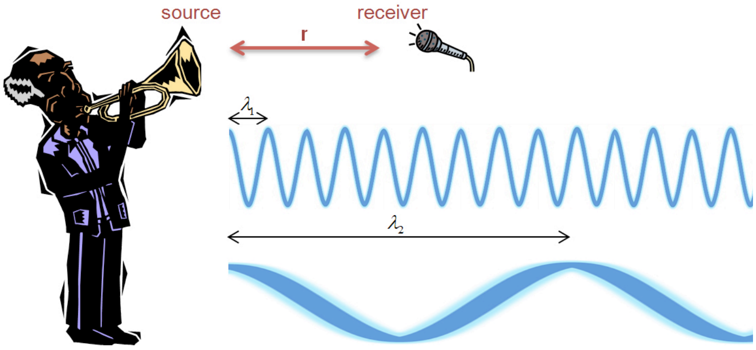


Fig.2 –Far and close field difference

By the example in *Figure 2* we note that the distance between source and receiver remains the same, however the field definition changes depending on the wavelength (in particular we get a far field if we consider the high frequency of the first wave and instead a near field considering the low frequency in the second wave).

So let's study the proprieties and effects of both fields:

- **Far field** ($r >> \lambda$): in this kind of field it's possible to suppose $k \cdot r >> 1$.
  Then we realize the term 1 in the velocity nominator (9) is insignificant compared to ***kr***. So computing and simplifying, the square of ***r*** in the denominator disappears. At the same time the relationship between pressure and ***r*** is not affected by the value of ***kr***.
  Of consequence the proportionality laws in far field are the following:

Far field

$$p \propto \frac{1}{r} \qquad v \propto \frac{1}{r} \qquad (10)$$

- **Near field**($r << \lambda$): as opposite of the first kind of field we get $k \cdot r << 1$.
  In this case, in the velocity nominator (9) the insignificant term will be kr and no simplification occurs. The term r^2 will still be in the

denominator. The relationship between pressure and distance will remain the same as before.
At last the **p** and **v** tend to these:

Near field

$$p \propto \frac{1}{r} \qquad v \propto \frac{1}{r^2} \qquad (11)$$

This means that close to the source the particle velocity becomes much larger than the sound pressure. Furthermore a close field will often occur for low frequencies. Then, this will be important related to the different kind of microphones which exist; the more a microphone is directive, as cardioids or hyper cardioids ones, the more it will be sensitive to the particle velocity, as opposite to an omnidirectional microphone which senses only the sound pressure. So, the more a microphone will be placed close to the source the more low frequencies will be boosted, and this is called proximity effect.

The capability of understanding and taking advantage of this effect could be really useful for singers.

Fig.3 –A singer who is "eating" the microphone

The more the microphone is placed far from the singer's mouth the more high frequencies will be captured (in proportion to low frequencies). This is the perfect situation for making high notes with voice. At the same time, in order to sing in a deeper way, the singer may decide to approach more and more the microphone, until giving the impression of "eating" it. And this boosts the low end of the spectrum of his voice.

Impedance

The impedance of a spherical field is a characteristic quantity, which is defined as the ratio of sound pressure and particle velocity. We can get the following expression of impedance using the solutions of D' Alembert equation:

$$Z(r) = \frac{p(r,\tau)}{v(r,\tau)} = \rho_0 \cdot c \cdot \frac{j\,kr}{1 + j\,kr} = \rho_0 \cdot c \cdot \left(\frac{k^2 \cdot r^2}{1 + k^2 \cdot r^2} + j \cdot \frac{k \cdot r}{1 + k^2 \cdot r^2} \right) \qquad (12)$$

Then it's possible to analyse this quantity far or close to the sound source. For the following cases let's consider this expression of impedance:

Impedance

$$Z(r) = \rho_0 \cdot c \cdot \frac{j\,kr}{1 + j\,kr} \qquad (13)$$

- **Far from the source**: as it has been already explained the distance has to be evaluated in comparison with the wavelength. The far field occurs when it's possible to suppose $r >> \lambda$ and of consequence $k \cdot r >> 1$.
  In the impedance's denominator the term 1 will be insignificant compared to $j\,kr$, and simplifying we get $Z(r) = \rho_0 \cdot c$. This represents the same impedance of a plane, progressive wave, as the imaginary part vanishes. This means that pressure and velocity are in phase far from the source: the more the wavefront travels away from the source, the larger will be its curvature radius, meaning that it actually behaves as a plane wave.
  This is the most usual case of wave propagation.

- **Close to the source**: again the distance must be compared to the wavelength $(r << \lambda)$ and we can suppose $k \cdot r << 1$.
  In this condition the denominator becomes substantially 1, so we get finally $Z(r) = \rho_0 \cdot c \cdot j\,kr$. Impedance is an imaginary number and of consequences velocity and pressure will have a phase shift of 90° (when the value of velocity is at maximum the value of pressure is null and vice-versa).

Then, for a little sphere in which $(R << \lambda)$, it will be difficult to radiate energy. The sound intensity is given by:

$$I = p_{rms} \cdot v_{rms} \cdot \cos(\varphi) \qquad (14)$$

If we consider now the phase shift φ becoming very close 90°, Intensity will tend to zero. This is a sort of paradox, in which, while this small sphere is pulsating with extremely large velocity, it seems to not irradiate any energy.
We conclude that a "little" sphere will be able to irradiate less energy than a bigger one.

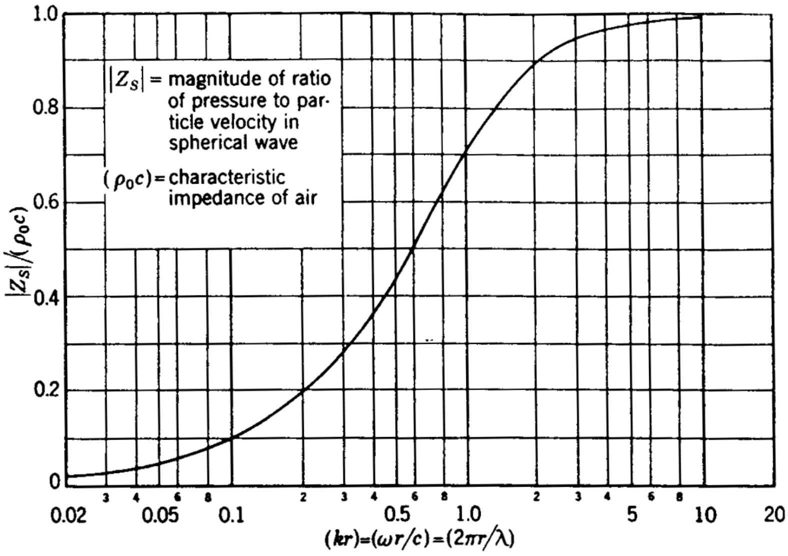


Fig.4 –Impedance (Magnitude)

Figure 4 shows us the chart of normalized impedance magnitude, given by the ratio of impedance's magnitude $|Z_s|$ and the characteristic impedance of air $\rho_0 \cdot c$, plotting in abscissa the value of kr and so either distance or frequency. When kr increases the curve of impedance boosts up. Above a value of $kr = 5$ energy transmission will be easier and easier. Oppositely, when kr becomes small, the impedance decreases significantly and energy transmission will be harder and harder.

It is possible to draw another chart as in *Figure 5*. In the abscissa we still have kr, while in ordinate we see the phase angle φ in degrees. So we can see here an explicit representation of the phase's variation and of what it implies. When kr decreases the phase shift tends to 90°, while if kr increases pressure and velocity result in phase. So it is clear that in the first case energy transmission will be hard because the Intensity tends to zero.

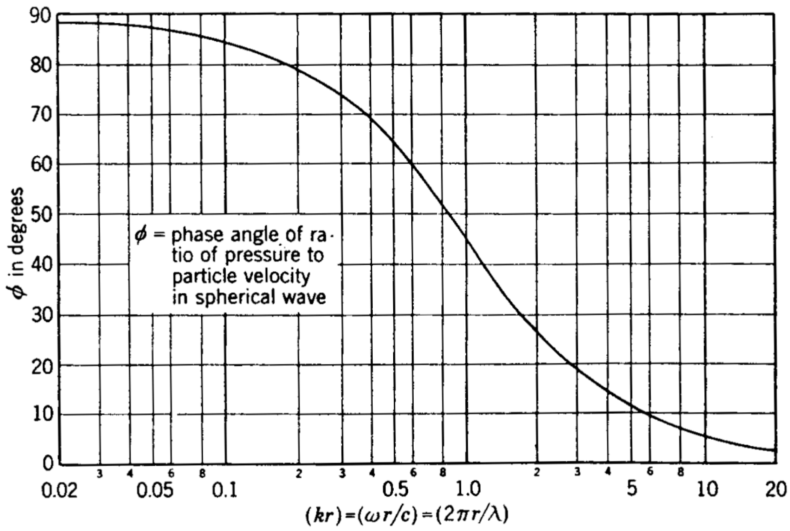


Fig.5 –Impedance (phase)

Energetic analysis and propagation law

Let's consider a far field, the most common one, in which sound pressure and particle velocity are in phase and an energetic analysis is easy.

We defined the sound intensity **I** as the ratio of sound power and surface:

Sound intensity

$$I = \frac{W}{S} \qquad (15)$$

Then, if we get a point source of power **W** , it's possible to note that a geometrical divergence occurs according to the increase of the distance.
In particular the area over which the power is dispersed increases with the square of the distance as shown in *Figure 6.*

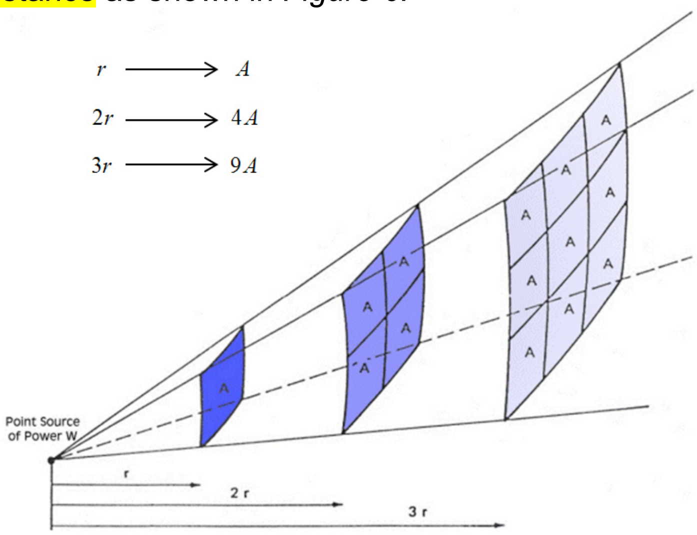


Fig.6 –Geometrical divergence

Now, remembering that a point source irradiates a spherical wave over a spherical surface **S,** we can write the following expression:

$$I = \frac{W}{S} = \frac{W}{4\pi r^2} \qquad (16)$$

Using this formula it is possible to find the equation which describes the spherical waves free propagation from an omnidirectional source (meaning that for every direction the same sound intensity is transmitted and that the space is free form any reflecting surfaces).
In order to find this equation let's compute the intensity level L_I , which is the representation in dB of the sound intensity:

$$L_I = 10\log\frac{I}{I_0} \qquad (dB) \qquad (17)$$

Employing the value of intensity as we see in formula (16) and multiplying and dividing for W_0 (in order to extract the power lever L_w) we have:

$$L_I = 10\log\frac{I}{I_0} = 10\log\left(\frac{\frac{W}{4\pi r^2}}{I_0}\right) = 10\log\left(\frac{\frac{W}{4\pi r^2}}{I_0}\cdot\frac{W_0}{W_0}\right) \quad (18)$$

Next, computing and reorganizing the terms in logarithm:

$$L_I = 10\log\frac{W_0}{W_0} + 10\log\frac{W_0}{I_0} + 10\log\frac{1}{4\pi} + 10\log r^{-2} \quad (19)$$

where:

- $10\log\frac{W_0}{W_0}$ is the power level L_w
- $10\log\frac{W_0}{I_0}$ is a null term. Indeed W_0 and I_0 have different measurement units but the same numerical value 10^{-12}. Of consequence $\log(1) = 0$
- $10\log\frac{1}{4\pi}$ is equal to -11
- $10\log r^{-2}$ can be written as $-20\log r$

Finally for a spherical and omnidirectional wave far from any reflecting plane the d' Alembert equation could be replaced with the following one, that is the free field propagation law in dB:

Propagation law in free field

$$L_I = L_w - 11 - 20\log r \qquad (dB) \quad (20)$$

Unfortunately a spherical wave can propagate free from any reflecting surface in a really few cases. Free field conditions can be obtained in a lab, inside an anechoic chamber, which is shown in *Figure 7.*

So, if the point source is near a reflecting surface it will be necessary to introduce a corrective factor.

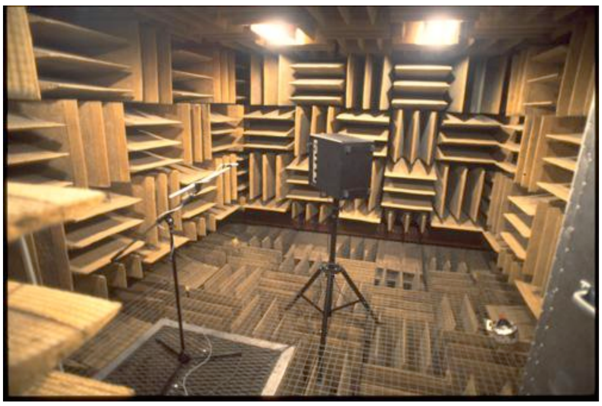


Fig.7 –An anechoic chamber

Directivity and general propagation law

As it has been said the formula (20) has limitations according to the position of the sound source

It's possible to introduce the directivity factor **Q**, which is the ratio of sound intensity in direction ϑ and the average sound intensity in the case of an omnidirectional source:

Directivity

$$Q = \frac{I_\vartheta}{I_0} \qquad (21)$$

Figure 8 shown us schematically the difference between the waves radiated by an omnidirectional source (blue) and by a generic one (red).

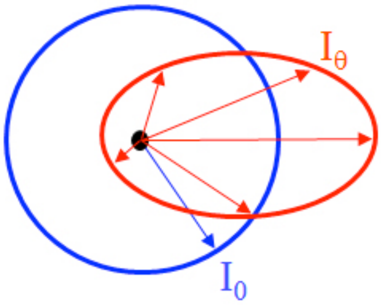


Fig.8 –Omnidirectional source (blue) and generic source (red)

So directivity will change according to the direction and to the frequency.
In order to write a new general propagation law the directivity factor will assume a logarithmic form.
Then we introduce the directivity index **DI**:

Directivity index

$$DI = 10 \cdot \log(Q) \qquad (dB) \qquad (22)$$

This index will be added to the formula (20).
In most cases we can assume to be in far field, in which case sound pressure and velocity are in phase and in this means that $L_p = L_I$ (to express the formula with L_p is much reasonable because of our ears, which sense pressure, not intensity).
At last the general propagation law will be written as the following one:

General propagation law

$$L_p = L_I = L_w - 11 - 20\log r + 10\log Q \qquad (dB) \qquad (23)$$

We even note that the pressure level changes according to the distance from the source. If it is doubled, having so $r \cdot 2$, the first logarithm in formula (23) will be:

$$-20\log(r\cdot 2) = -20\log(r) - 20\log(2) = -20\log(r) - 6 \qquad dB\ .$$

This means that for spherical waves every time the distance is doubled the value of L_p decreases by $6dB$.

This value is called decay factor ΔL_2 and it will change according to the different kind of waves (for cylindrical waves the decay factor is 3 dB instead of 6).

Finally it's possible to study the directivity factor when a point source is placed near one or more reflecting surfaces. We can for example differentiate four cases, as shown in *Figure 9*.

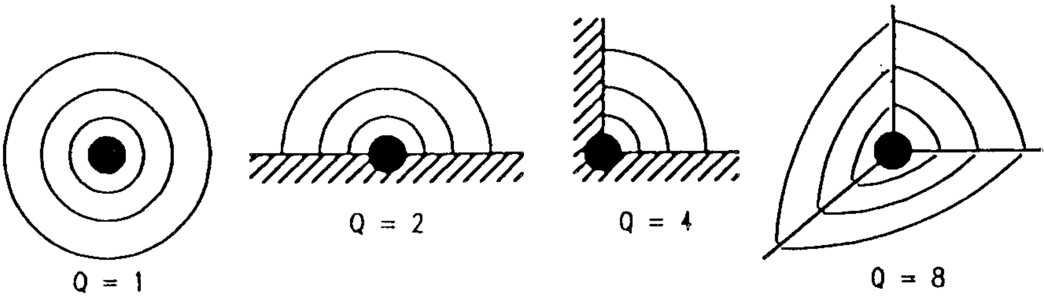


Fig.9 –Q value according to its position close to reflecting surfaces

We differentiate these four cases:

- $Q = 1$ when the source is far from any reflecting surface. In this case it will be possible to use formula (20), in which the directivity index disappears: $DI = 10 \cdot \log(Q) = 10 \cdot \log(1) = 0$
- $Q = 2$ when the source is over a reflecting surface. Above the reflecting plane the intensity will disperse on a hemisphere and not on a sphere. So its value will be doubled because of reflection.
- $Q = 4$ when the source is in a corner. The intensity will be dispersed on a quarter of sphere, so its value will be 4 times the free field intensity.
- $Q = 8$ when the source is in vertex. As the previous cases the intensity grows of factor of eight.

CYLINDRICAL FIELD

Differently from point sources, line sources (roads, railways, airtracks, etc.) are characterized by a cylindrical field.

Through this model, you can consider single sources moving in time on a linear trajectory (for example: cars on the road) as a continuous event, that disperse the total power of the sound over a cylindrical surface.

This equality allows an easier approach to acoustic problems.

The cause-effect relationship giving the SPL value in a cilyndrical field is represented by two different equations:

$$L_p = L_{W'} - 10\log d - 6 \qquad (incoherent \quad emission)$$
$$L_p = L_{W'} - 10\log d - 8 \qquad (coherent \quad emission)$$

Where **Lw'** is the sound power level per meter of a line source.

The first formula is the most used in acoustic problems, because it refers to concrete situations.

The second one refers to an unreal model (the pulsating cylinder: sound is always the same in all directions and in all points of the space), but thanks to the semplicity of the situation it represents, it is easier to prove and apply.

COHERENT CYLINDRICAL FIELD

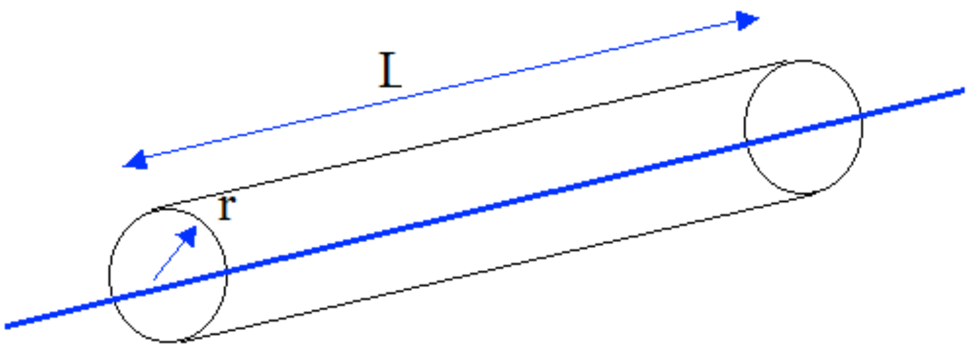


Fig.10 – a pulsating cylindrical source

Considering an infinitely long (L>>r), pulsating cylinder, you obtain the following formulas:

$$I = \frac{W}{S} = \frac{W}{2 \cdot \pi \cdot r \cdot L}$$

$$L_I = 10 \cdot \lg\left[\frac{I}{I_o}\right] = 10 \cdot \lg\left[\frac{\frac{W}{2 \cdot \pi \cdot r \cdot L}}{I_o}\right] = 10 \cdot \lg\left[\frac{\frac{W}{2 \cdot \pi \cdot r \cdot L}}{I_o} \cdot \frac{W_o}{W_o}\right] = 10 \cdot \lg\left[\frac{W}{L \cdot W_o}\right] - 10 \cdot \lg[2 \cdot \pi] - 10 \cdot \lg[r]$$

$$L_I = L_W' - 8 - 10 \cdot \lg[r]$$

(coherent emission)

Where

I: sound intensity at distance **r** by the source.
W: sound power let out by the source.
S: cylindrical surface.
L_W': sound power level per meter of line source
-10·log[r]: how sound power decreases with distance.

Comparing spherical and cylindrical waves, you notice an important difference: the cylindrical waves are less sensitive with distance.
In fact

SPHERICAL FIELD →	pressure level reduces by 6 dB for doubling distance **r** (because of the term –**20log[r]**)
CYLINDRICAL FIELD →	pressure level reduces by 3 dB for doubling distance **r** (because of the term –**10log[r]**)

DISCRETE AND INCOHERENT LINEAR SOURCES

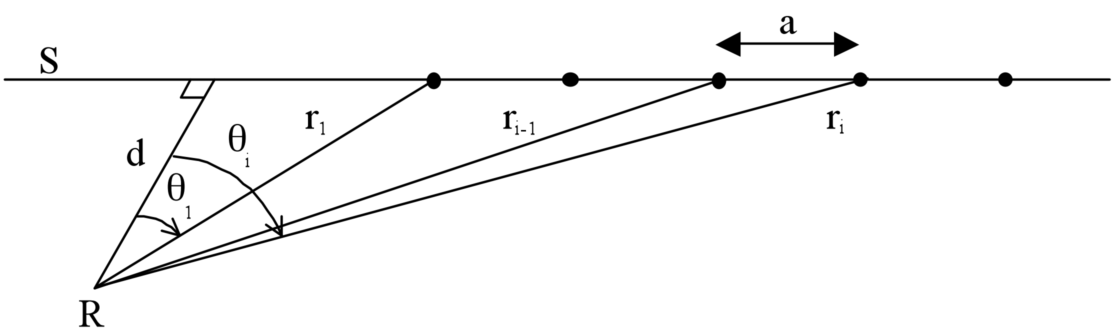


Fig.10 – line source made of a number of point sources

This figure is an example of linear source made of a number of point sources.

The horizontal line can be imagined as a road, and the single points as cars at distance **a**. The minimum distance between the receiver and the line source is **d**.

In reality, the single cars are incoherent point sources: the sounds they produce are not in phase and not correlated.

Approaching a problem as a continuous event, you can study this example and apply equations of the cylindrical field.

The average car-to-car distance, **a**, can be computed as:

$$a = V / N \cdot 1000 \quad [m]$$

The sound power level of each car, $\mathbf{L_{Wp}}$, is consequently distributed along a piece of road with a length of **a** meters, providing a power level per meter, $\mathbf{L_W'}$, given **by $\mathbf{L_{Wp}}$ - 10log(a)**. Hence, the cause-effect relationship becomes:

$$L_p = L_{Wp} - 10\log(a) - 10\log(d) - 6 \quad [dB]$$

(incoherent emission)

Where:

$\mathbf{L_{Wp}}$-10log(a) = $\mathbf{L_W'}$: sound power level per meter. It depends on the power level of a single vehicle and the distance between vehicles.

$\mathbf{L_{Wp}}$: power level of a single vehicle.

-10log(d): decrement of sound with distance. This term depends on the position of the receiver.

V: speed of the vehicle [km/h].

N: number of vehicles passing in one hour [number of vehicles/h].

The sound power perceived by a listener depends on the distance **d** from the linear source and the sound power level per meter.

$\mathbf{L_{Wp}}$ is fuction of the speed : it increases with speed.

For low speed the sound power of a single vehicle changes a little, but for high speed it changes with the square of speed.

- up to 50 km/h → $\mathbf{L_{Wp}}$ is constant
- between 50 km/h and 100 km/h → $\mathbf{L_{Wp}}$ increases linearly with **V** (3dB/doubling)
- above 100 km/h → $\mathbf{L_{Wp}}$ increases with $\mathbf{V^2}$ (6dB/doubling)

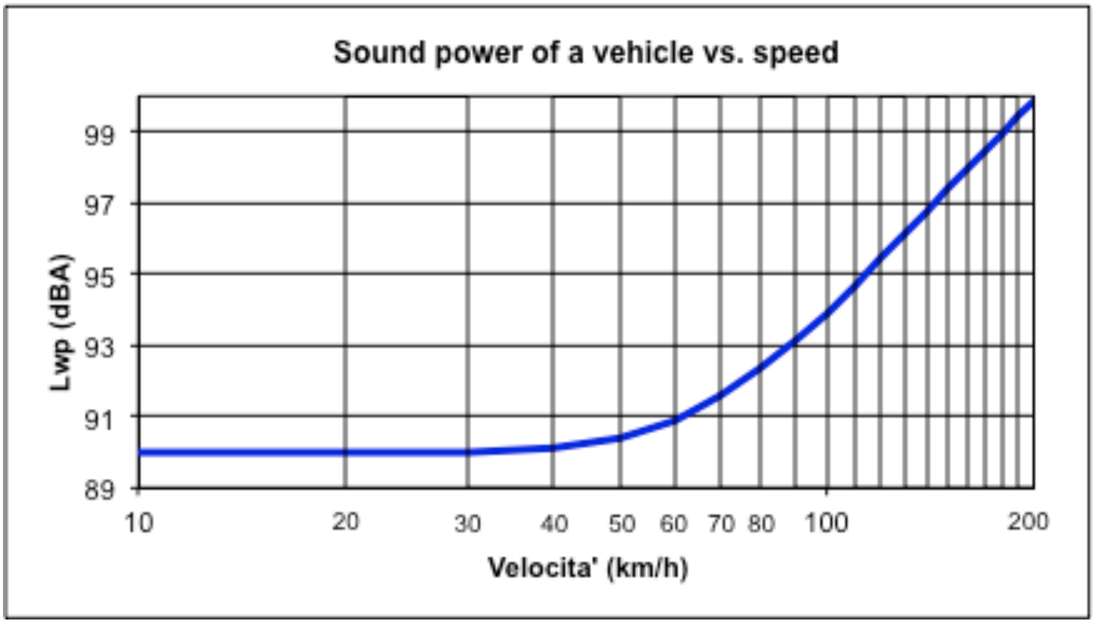


Fig.11 – sound power level vs. speed

There is an important connection between the speed, the distance **a** and the power level: higher speed means bigger distance and power of a single vehicle **L_{Wp}**, but it is not the same for the sound power level per meter **L_W'**.

In fact it increases with **L_{Wp}**, but it also decreases the vehicle-to-vehicle distance **a**. For this reason, the link between power level per meter and speed is non-linear.

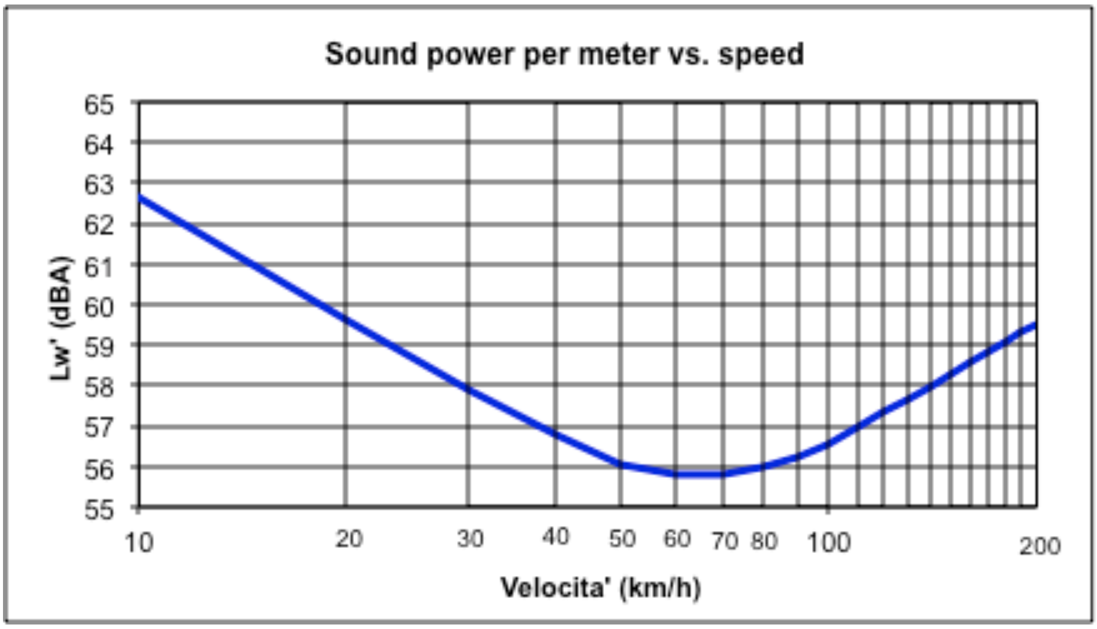


Fig.12 – sound immission level vs. speed

The best combination between **Lwp** and distance **a** gives us the "optimal" speed, about 70 km/h. It causes the minimum value of **SPL** (and consequently less noise pollution).

Due to technological innovations the value of the "optimal speed" is becoming even larger: the features of new cars make vehicles less noisy up to higher speeds, where the aerodynamic noise becomes predominant.

EXCESS ATTENUATION

In reality, between source and receiver ,there are often obstacles and other factors, which cause additional attenuation of sound level.

These factors are mainly:

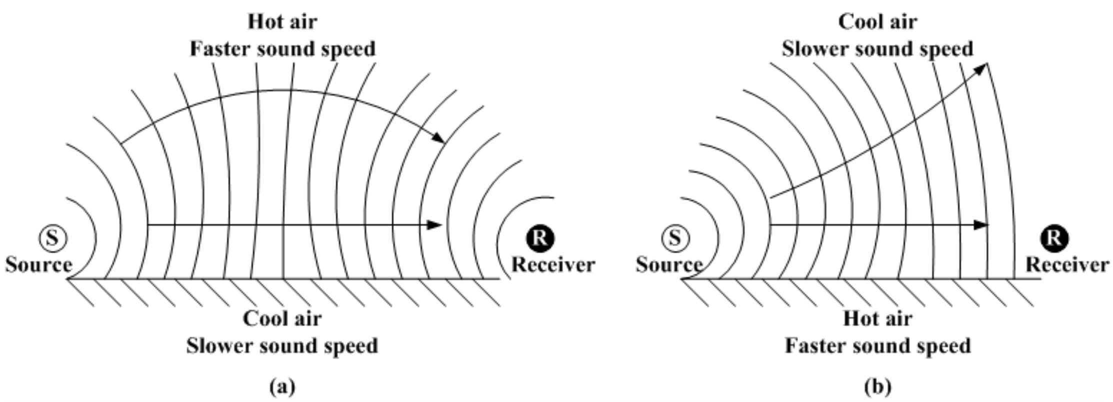
1) air absorption
2) absorption due to presence of vegetation, foliage etc.
3) meteorological conditions (temperature gradients, wind speed gradients, rain, snow, fog, etc.)
4) obstacles (hills, buildings, noise barriers, etc.)

The term **ΔL**, in the spherical free field formula, quantifies all these factors :

• $L_I = L_p = L_W - 20 \log r - 11 + 10 \log Q - \Delta L$ (dB)

Most of these effects are relevant only at large distance from the source and at high frequency. The exception in shielding (maximum when the receiver is very close to the screen).

TEMPERATURE GRADIENT



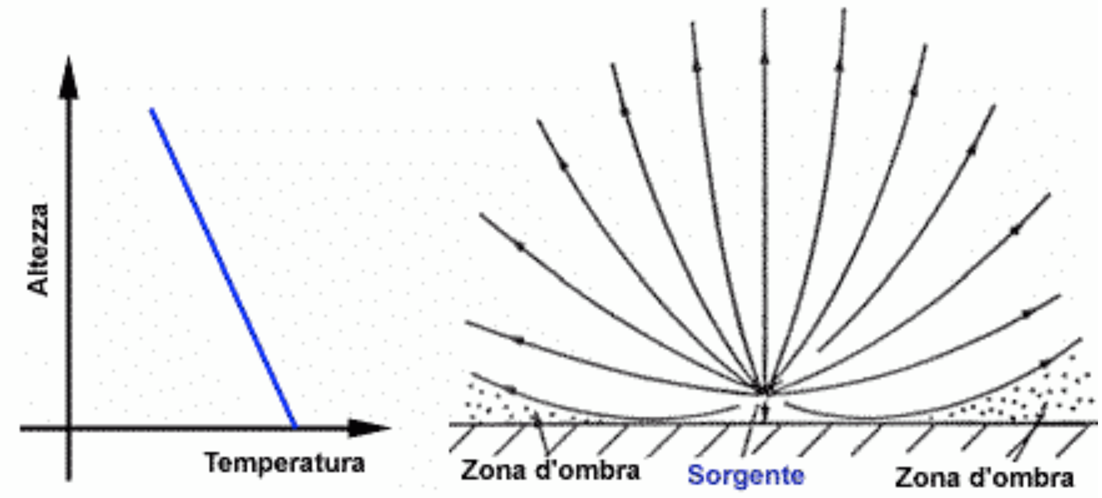


Fig.13 – effects of temperature gradient

Sound speed depends on temperature gradient ⇒

Temperature decreases with height →	- sound speed decreases too - sound rays curve upward - on the ground, at a certain distance from the source you find the shadow zone (there is no sound)
Temperature increases with height →	- this is a particular phenomenon caused by the fog layer above the ground - sound rays curve downward - due to their particular curvature, sound rays can be perceived far from the source

WIND SPEED GRADIENT

In this case, the attenuation is not caused by the "transport" of the sound due to the wind. In fact, wind speed is very low in comparison with sound speed (340 m/s).

The real effect produced is the curvature of sound rays caused by wind gradient (the speed increases with height), similarly to temperature.

Downwind → sound rays curve downward and jump over obstacles

Windward → sound rays curve upward causing the shadow zone and strong attenuation

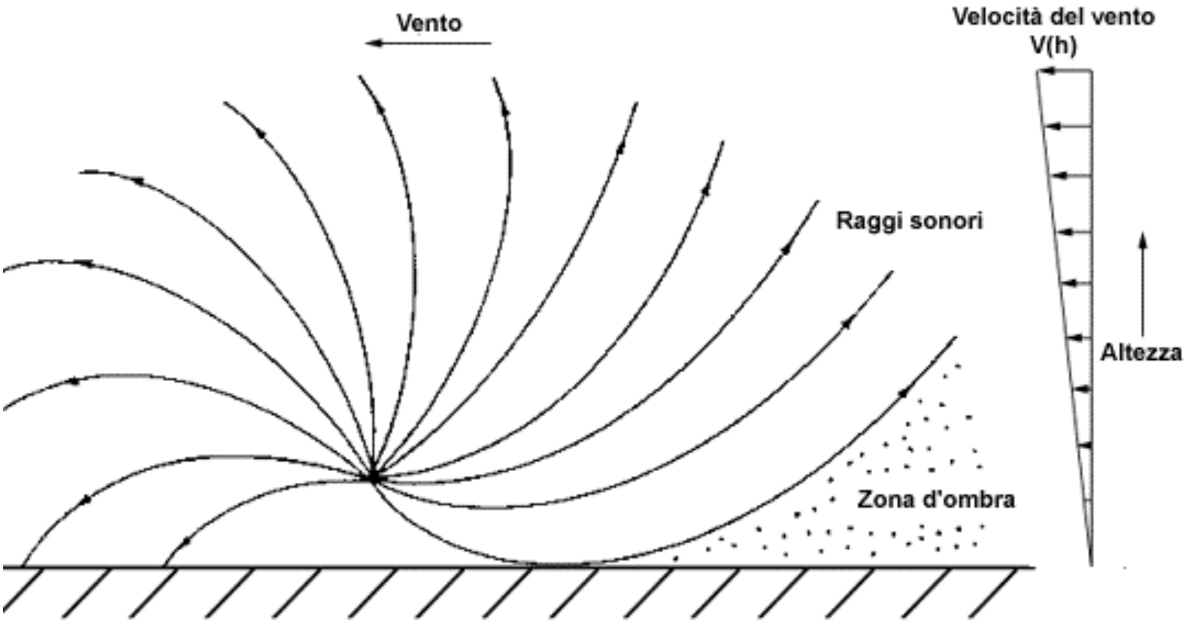


Fig.14 – effects of wind speed gradient

AIR ABSORPTION

		Frequency (octave bands)							
T (°C)	***RH (%)***	***63***	***125***	***250***	***500***	***1000***	***2000***	***4000***	***8000***
10	70	0,12	0,41	1,04	1,93	3,66	9,66	32,8	117,0
15	20	0,27	0,65	1,22	2,70	8,17	28,2	88,8	202,0
15	50	0,14	0,48	1,22	2,24	4,16	10,8	36,2	129,0
15	80	0,09	0,34	1,07	2,40	4,15	8,31	23,7	82,8
20	70	0,09	0,34	1,13	2,80	4,98	9,02	22,9	76,6
30	70	0,07	0,26	0,96	3,14	7,41	12,7	23,1	59,3

The table above shows air absorption coefficients [dB/km] for different combinations of frequency, temperature and humidity.

You can notice that:

- dry air causes more absorption than damp air
- air absorption increases with frequency (low frequencies travel easily; high frequencies are rapidly absorbed by air)

Noise screens

Commonly, a noise barrier is a solid structure, positioned between the source and the receiver, which blocks the direct path of sound propagation.

Whenever a noise screen is positioned between the source and the receiver, it provides a reduction of the sound pressure level, and it can even eliminate it completely in those particular spaces called "shadow zones" where the sound does not travel at all.

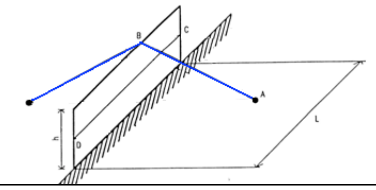
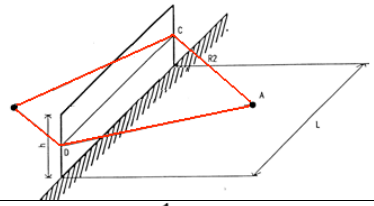
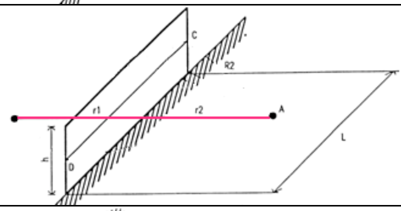
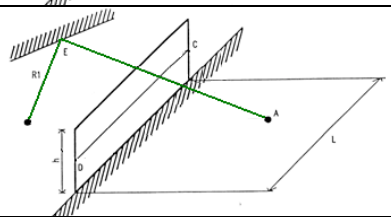
To ensure an easier picture of the concept the noise barriers will be considered as thin walls.

The acoustic efficiency of a noise screen is represented by the insertion loss ΔL

$$\Delta L = L_0 - L_b \qquad (dB)$$

Where L_0 and L_b are the SPL values with and without the screen.

In most situations there are several different paths a sound can follow to reach the receiver when the barrier is installed:

Diffraction at upper of the screen: i.e. the barrier is very long and the effective heights not too high	
Diffraction at sides of the screen: when the distance between the source and the screen edge is less than 5 times the effective height	
Passing through the screen	
Reflection over other surfaces present in proximity: when the location allows it	

In this class we will deal only with the first two cases.

Noise screens : the Maekawa formulas

Depending on the case, we have to use different mathematical formulas to obtain ΔL.

In this class we are going to explain the different formulas for solve the first two abovementioned cases.

If we only consider the energy diffracted by the upper edge of a thin, infinitely long barrier we can estimate the insertion loss as:

$\Delta L = 10\log(3+20N)$	for N>0 (for spherical field)
$\Delta L = 10\log(2+5.5N)$	for N>0 (for cylindrical field)

We can see that in these formulas, called Maekawa's formulas ΔL is function of the number N, which is the Fresnel number defined by:

$$N = \frac{2\delta}{\lambda}$$

δ = difference between the diffracted path and the directed path.
It is always > 1 if there is a diffraction.
λ = wavelength

The Maekawa formulas are logarithmic formulas, so we obtain a result expressed in decibel (dB).

We can also express the Fresnel number N as function of the frequency because:

$$N = \frac{2\delta}{\lambda} = \frac{2f\delta}{c} \quad \textit{because} \quad \lambda = \frac{c}{f}$$

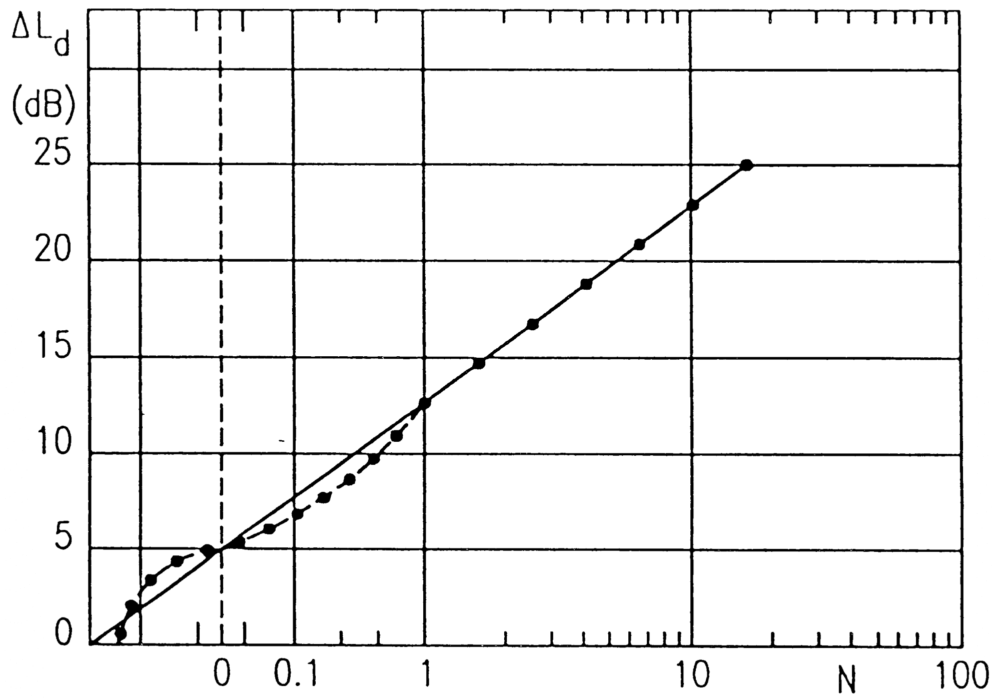
f = frequency
c = sound speed

Maekawa chart

If we want to tabulate the results in order to see the relationship between the Fresnel number N and the insertion loss, we can use the Maekawa chart.

The line that describes that relationship is different depending on the case we are studying.

If we are studying a spherical field (generated by a point source) the Maekawa chart is the following:



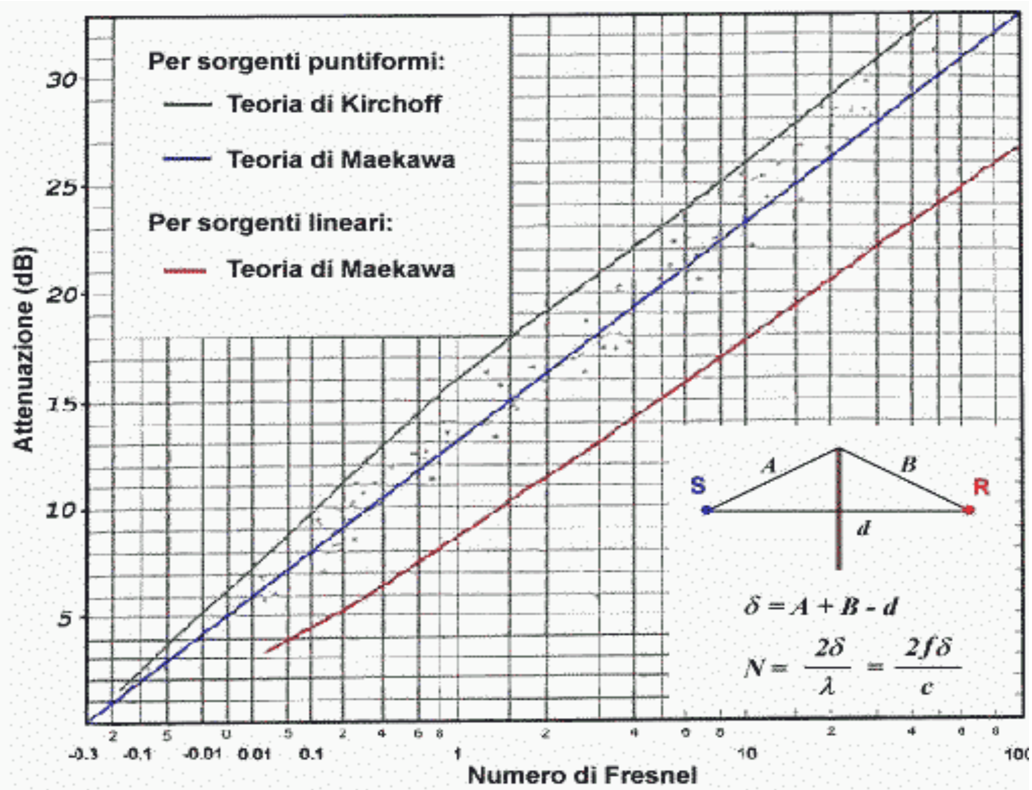
In the graphic, the straight, solid line represents the Maekawa formula, while the curved, dotted line represents the pattern obtained by connecting the results of actual measurements.

When the Fresnel number is larger than 1, these two lines coincide.

For N<1, instead, the Maekawa formula causes some errors: for better accuracy, it is recommended to employ the Kurze-Anderson formula (which indeed provides accurate results also for N>1):

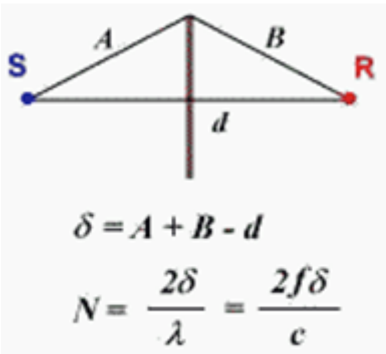
$$\Delta L = 5 + 20 \cdot \log\left[\frac{\sqrt{2\cdot\pi\cdot N}}{\tanh\left(\sqrt{2\cdot\pi\cdot N}\right)}\right]$$

The following chart represents the relationship between the Fresnel number N and the insertion loss ΔL for spherical fields and cylindrical fields.



In this chart the blue line represents the Maekawa formula trend for spherical fields; the red line represents the Maekawa formula for cylindrical fields, and last of all, the gray line represents the trend of the Kirchoff formula, another formula to obtain the relationship between N and ΔL and which behaves quite poorly in real world.

At the bottom of the chart we can see a figure that clarifies some concepts.



First of all we can see the meaning of δ: δ is the path difference among the diffracted and the direct sound.

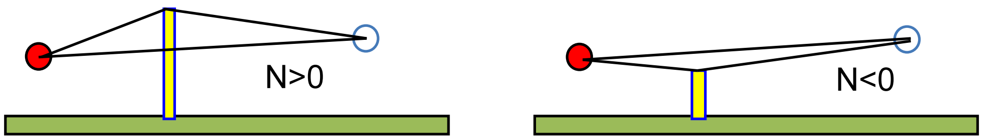
We can also give a definition to the term effective height.

Effective height h_{eff}: it is a length, expressed in meters, that represents the distance between the upper border of the barrier and the intersection with the barrier of the line connecting the source and the receiver.

h_{eff} is not the same height as the barrier's height, because if source and receiver are not on the ground, or if the ground is not horizontal, the effective height is systematically smaller than the total height of the barrier. This concept can be better understood thinking that the barrier is divided in two parts: the lower one reaches the "see-not-see" height, at which the receiver can barely see the source through a line-of-sight which is tangent to the screen. All the further elevation of the barrier above this “line of sight” becomes the effective height of the barrier.

One could think that, for getting any effective sound attenuation, the barrier must be taller than the “line of sight”, and if the barrier is too short, allowing the receiver to “see” the source, there is no attenuation.

However, adopting the convention that the Fresnel number is positive whenever h_{eff} is positive, and that N becomes negative when h_{eff} is negative (that is, when the barrier is below the line-of-sight), we see that the Maekawa formulas still work, as shown in the charts above, if N > -0.3



And this means that some small sound reduction is obtained even when installing a noise screen, which appears too short, as it allows the receiver to “see” the source.

When the value of N=0 (that is, when the height of the barrier is tangent to the “line of sight”), we still get a significant attenuation, which is equal to 5 dB for a point source and 3.0 dB for a line source, as shown by the Kurze-Anderson formula and by the Maekawa chart.

Noise screens: finite length

If the length of the barrier is not long enough, we must also consider its lateral edges.

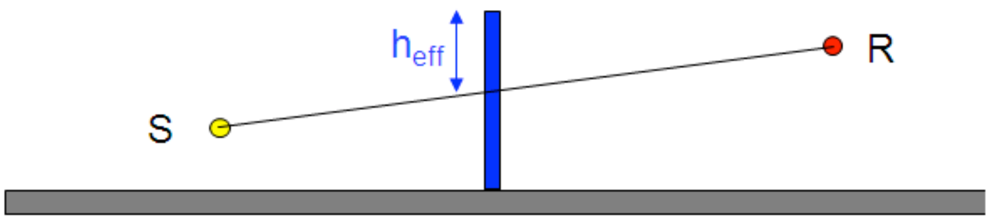
Each edge has its own Fresnel number (N_1, N_2) and this causes a change in the formula relating the Fresnel number with the insertion loss.

The new formula is:

$$\Delta L = \Delta L_b - 10\log(1 + \frac{N}{N_1} + \frac{N}{N_2})$$

Valid for values of $N,N_1,N_2 > 1$ – in this formula ΔL_b is the insertion loss caused by the upper edge, evaluated with the Maekawa or Kurze-Anderson formulas.

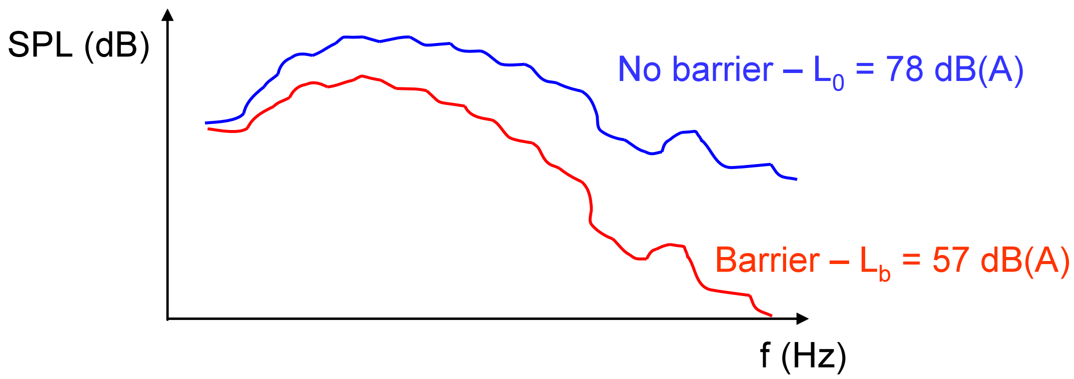
We have to use this formula whenever the side edge is closer than 5 times the effective height.



The insertion loss changes with frequency: hence we have to draw the noise spectrum twice, both in absence and in presence of the barrier.

It is observed that the insertion loss increases with frequency.

In the figure below we can see the noise spectrum without the barrier (in blue) and the one with the barrier (in red).



For evaluating the total, A-weighted sound reduction caused by a barrier in dB(A), we have to compute ΔL separately at every frequency (for example in octave bands).

Then we compute the total A-weighted SPL for the spectrum before the installation of the barrier, we recompute the new total A-weighted SPL after the installation of the barrier, applying at every frequency the proper value of ΔL, and finally we get the A-weighted insertion loss, ΔL_A, as the difference between the two A-weighted values of total SPL, before and after the barrier installation.

$$\Delta L_a = L_{Abefore} - L_{Aafter}$$

This means that, whilst the values of ΔL at single frequencies depends only on the geometry, the total insertion loss in dB(A) also depends on the spectrum of the noise source.
And, as shown in the figure above, the value of ΔL increases with frequency. Hence, the very same barrier will provide a large reduction of the A-weighted noise level for sound sources having a spectrum that contains a lot of energy at high frequencies (example: train), and a significantly smaller reduction of the A-weighted SPL for noise sources having most energy at low frequencies (I large Diesel engines, trucks, highways, etc.).